

Exercise 43

Use logarithmic differentiation to find the derivative of the function.

$$y = x^x$$

Solution

Take the natural logarithm of both sides and use the properties of logarithms to simplify the right side.

$$\begin{aligned}\ln y &= \ln x^x \\ &= x \ln x\end{aligned}$$

Differentiate both sides with respect to x .

$$\begin{aligned}\frac{d}{dx}(\ln y) &= \frac{d}{dx}(x \ln x) \\ \frac{1}{y} \cdot \frac{d}{dx}(y) &= \left[\frac{d}{dx}(x) \right] \ln x + x \left[\frac{d}{dx}(\ln x) \right] \\ \frac{1}{y} \cdot \frac{dy}{dx} &= (1) \ln x + x \left(\frac{1}{x} \right) \\ \frac{1}{y} \frac{dy}{dx} &= \ln x + 1 \\ \frac{dy}{dx} &= y(\ln x + 1) \\ &= x^x(\ln x + 1)\end{aligned}$$